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Algebra 1

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Midterm

No credit will be given to unjustified answers. Justify all your answers completely. (Either with a proof or with a counter example) unless mentioned differently. No step should be a mystery or bring a question. The grader cannot be expected to work his way through a sprawling mess of identities presented without a coherent narrative through line. If he can't make sense of it in finite time you could loose serious points. Coherent, readable exposition of your work is half the job in mathematics. You will lose serious points if your exposition is messy, incomplete, uses mathematical symbols not adapted...

Problem 1 :
Let $\mathbb{C}^\times$ be the multiplicative group of nonzero complex numbers. Let $\mathbb{P}$ be the set of positive real numbers. Let $\mathbb{S}$ be the multiplicative group of all complex numbers with absolute value 1.

1. Prove that $\mathbb{P}$ is a subgroup of $\mathbb{C}^\times$.
2. Prove that $\mathbb{S}$ is a subgroup of $\mathbb{C}^\times$.
3. Let
$$\begin{array}{rccc} \tilde{c}: & \mathbb{C}^\times/\mathbb{P} & \to & \mathbb{S} \\ & z\mathbb{P} & \mapsto & \frac{z}{|z|} \end{array}$$
where $|z|$ is the modulus of $z \in \mathbb{C}$.
   (a) Prove that $\tilde{c}$ is well defined. That is, if $z\mathbb{P} = z'\mathbb{P}$, prove that $\frac{z}{|z|} = \frac{z'}{|z'|}$.
   (b) Prove that $\tilde{c}$ is a homomorphism.
   (c) Compute the kernel of $\tilde{c}$. Is $\tilde{c}$ one-to-one ?
   (d) Compute the range of $\tilde{c}$. Is $\tilde{c}$ onto $\mathbb{S}$?
   (e) Is it an isomorphism ?

Problem 2 :

1. Give an exhausted list of the element of S_3.
2. Is S_3 abelian ? Justify.
3. Is S_3 cyclic ? Justify.
4. Give all the elements of order 2 in S_3. What about elements of order 4 in S_3 ?
5. Let $H = <(1,2)>$
   (a) Describe H. How many elements are there in H ? To which well known group is H isomorphic to ? (Hint : Think about the classification of cyclic group.)
   (b) Describe the left and the right coset of the permutation $(1,2,3)$ for H. Is H a normal subgroup of S_3 ? Is S_3/H a group ?
6. Let $H' = \{e, (1,3,2), (1,2,3)\}$.
   (a) Doing the table of H', prove that H' is a subgroup of S_3.
   (b) Is H' cyclic ? If yes, give a generator of H'. To which well known group is H' isomorphic to ? (Hint : Think about the classification of cyclic group.)
   (c) Describe the element of the quotient space S_3/H'. How many distinct elements are there in S_3/H' ?
   (d) Let $\sigma \in H'$, we define

$$\begin{array}{cccc} \phi_\sigma : & H' & \to & H' \\ & f & \mapsto & \sigma \circ f \end{array}$$

   Show that ϕ_σ is a bijection by finding its inverse. Is ϕ_σ a homomorphism ?

   (e) Denote $Per(H')$ be the set of all the bijection of H' onto H'. It is a group with the composition operation. Let

$$\begin{array}{cccc} \Phi : & H' & \to & Per(H') \\ & \sigma & \mapsto & \phi_\sigma \end{array}$$

   i. Prove that Φ is a homomorphism.
   ii. Compute the kernel. Is Φ one-to-one ?
   iii. Compute the range. Is Φ onto $Per(H')$?
   iv. Is Φ an isomorphism ?